Full Factorial Design of Experiment (DOE)

(Six Sigma Green Belt Training Exercise)

prepared

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Minitab Solution to DOE GB Training Exercise

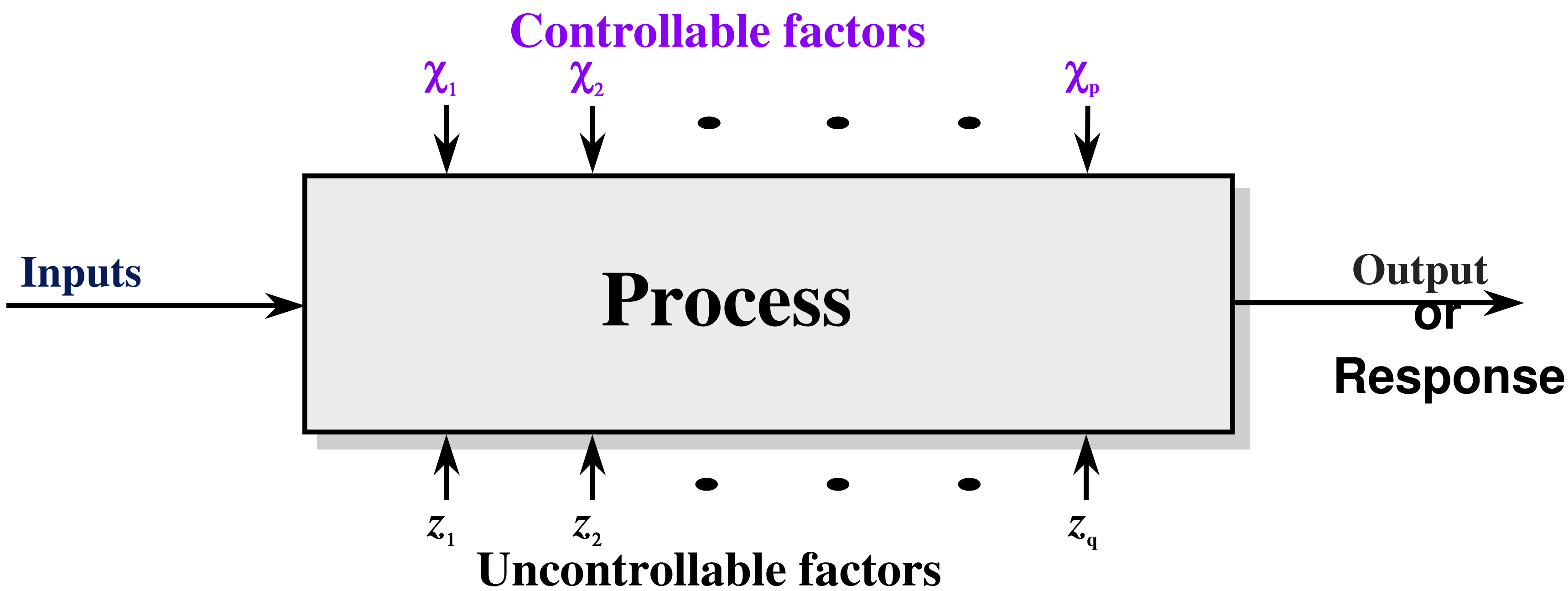
- The objective is to share Minitab solution of DOE performed during training on 3/10/03.
- The experiment was a 2-level, 3 factors full factorial DOE.

Factors

X_1 = Car Type
X_2 = Launch Height
X_3 = Track Configuration

Other

- The data is this analysis was taken from Team #4 Training from 3/10/2003.
- Please see Full Factorial Design of experiment hand-out from training.
- Please see pages 18 - 20 for an explanation and illustration on test for significance.



$$y = f(x_1, x_2, x_3)$$

Factors: A factor is one of the controlled or uncontrolled variables whose influence upon request is being studied in the experiment. A factor may be quantitative, e.g., temperature in degrees, time in seconds. A factor may also be qualitative, e.g., different machines, different operator, clean or no clean.

Factors and Settings

Factors	Settings	
X1: Car Type	(-) = Car #1	(+) = Car #2
X2: Launch Height	(-) = Chair	(+) = Box Top
X3: Track Configuration	(-) = No Bump	(+) = Bump

Design Matrix Created by Minitab

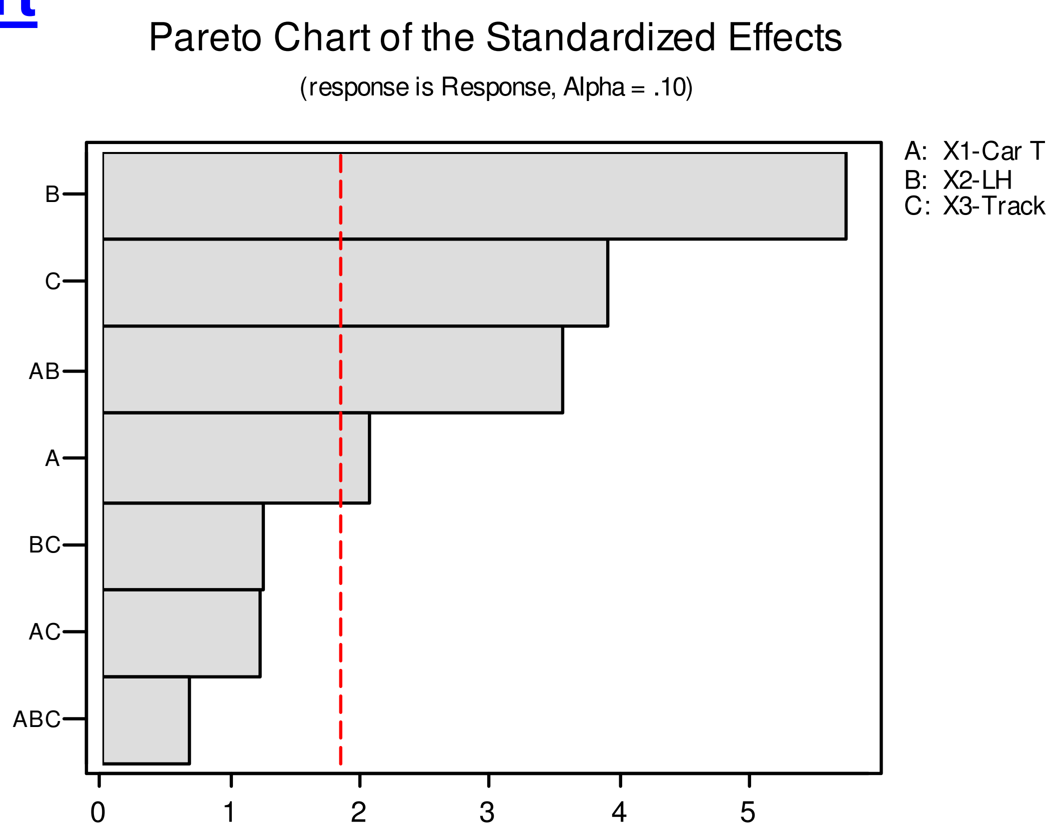
StdOrder	RunOrder	CenterPt	Blocks	X1-Car Type	X2-LH	X3-Track Conf	Response-Y
10	1	1	1	1	-1	-1	1.75
15	2	1	1	-1	1	1	1.68
12	3	1	1	1	1	-1	1.41
2	4	1	1	1	-1	-1	1.59
6	5	1	1	1	-1	1	1.94
9	6	1	1	-1	-1	-1	2.18
7	7	1	1	-1	1	1	1.59
11	8	1	1	-1	1	-1	1.38
14	9	1	1	1	-1	1	1.88
13	10	1	1	-1	-1	1	2.03
5	11	1	1	-1	-1	1	2.56
16	12	1	1	1	1	1	1.81
1	13	1	1	-1	-1	-1	2.09
4	14	1	1	1	1	-1	1.31
8	15	1	1	1	1	1	1.97
3	16	1	1	-1	1	-1	1.4

Design Matrix Created by Minitab

X1-Car	T*X2-LH	*X3-Track	Mean(avg.)	Std. Error of mean
-1	-1	-1	2.135	0.10640
1	-1	-1	1.670	0.10640
-1	1	-1	1.390	0.10640
1	1	-1	1.360	0.10640
-1	-1	1	2.295	0.10640
1	-1	1	1.910	0.10640
-1	1	1	1.635	0.10640
1	1	1	1.890	0.10640

Recall: These values were computed in class.

Pareto Chart

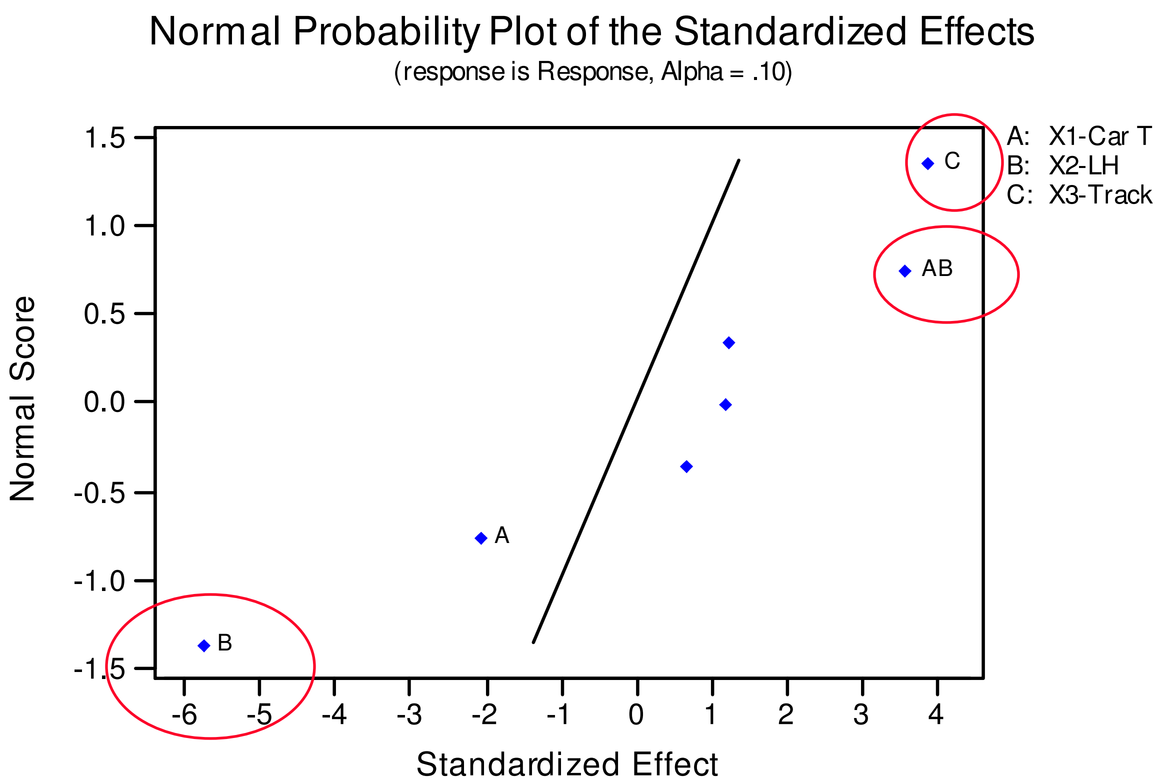


Comment:

Note that B, C, AB & A Pareto bars are to the right of the vertical red line; therefore, these bars represent statistically significant at the 10% level of significance. See analytical solution in table 2. The analysis in table 2 agrees nicely with this graph.

Normal Plot

(continued)

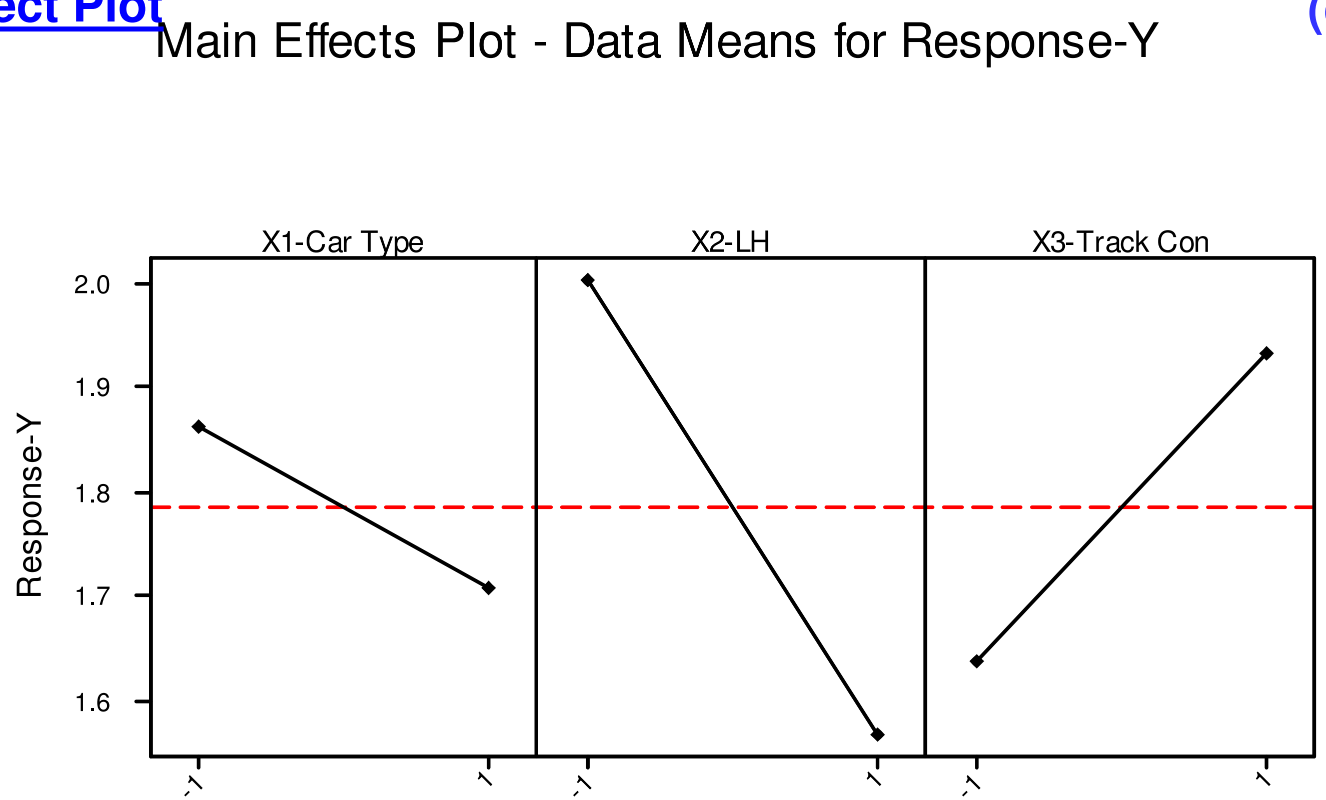


Comment:

Note that B, C, A & AB locations are again indicative of statistically significant.

Mean Effect Plot

(continued)

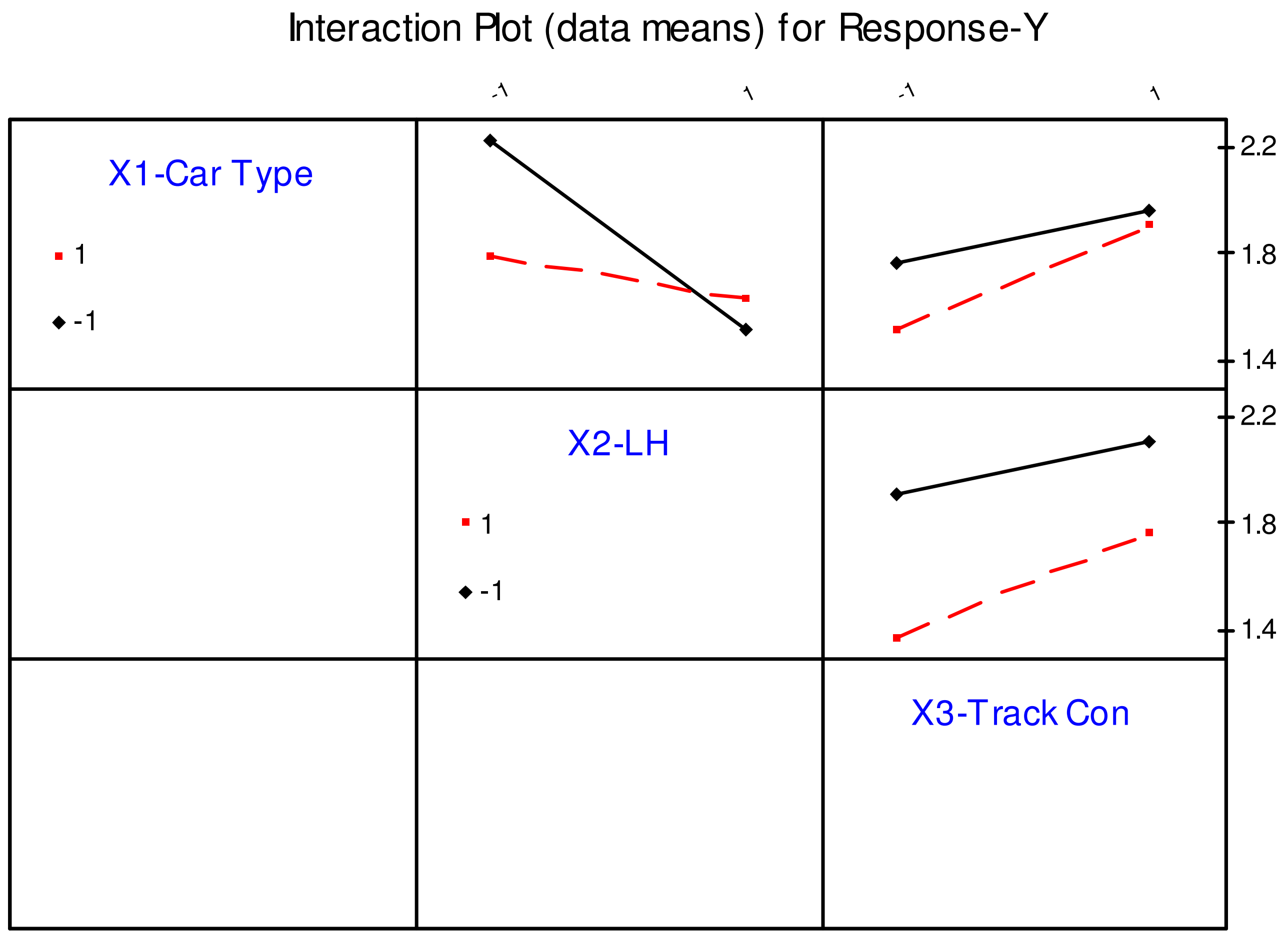


Comment

This plot should agree with sketch completed during training. Note that the slopes for x_1 and x_2 factors are downward (negative slope) which are consistent with their negative mean effects as computed in class.

Interaction Plot

(continued)



Indicates interaction between car type & launch height

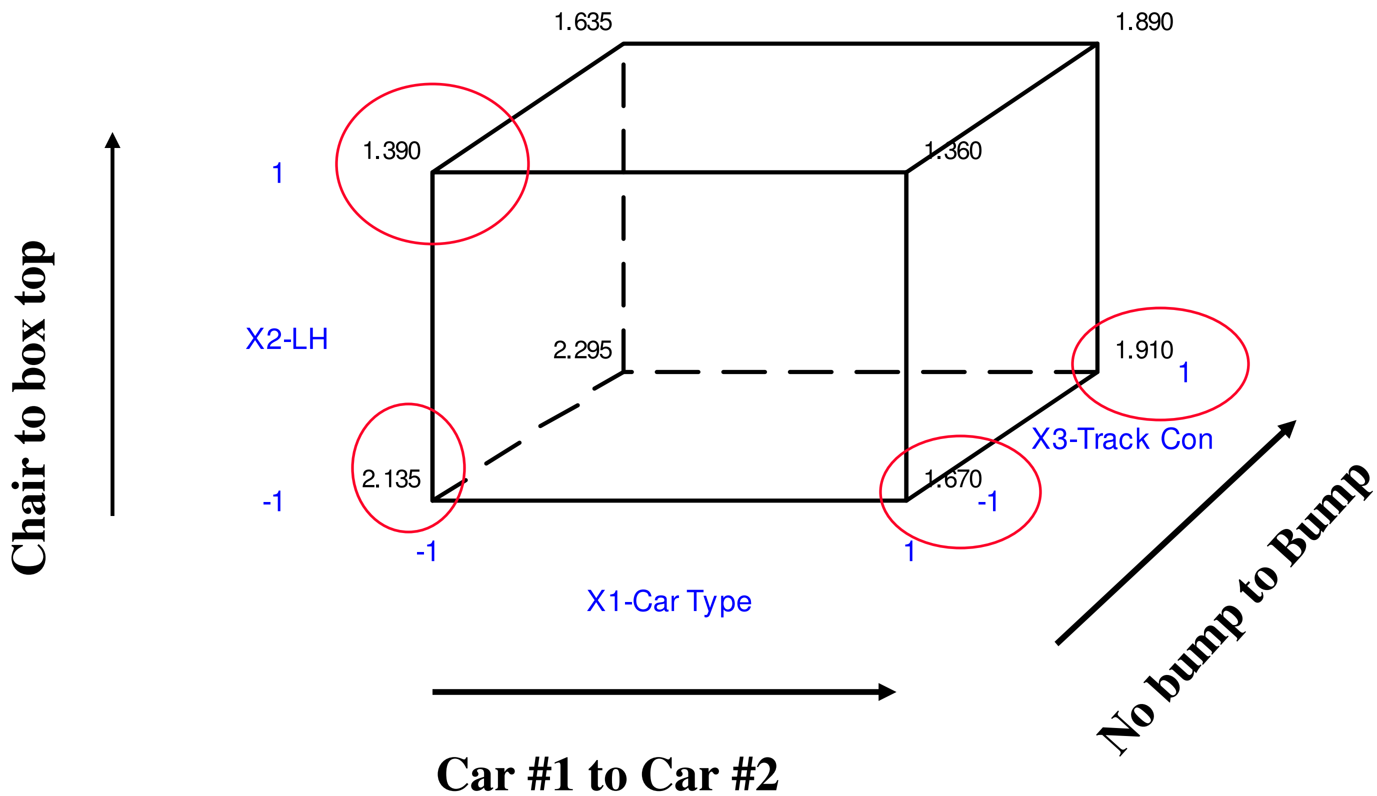
No interaction

Cube Plot

(continued)

Cube Plot (data means) for Response-Y

Note: If you take look at average value from the design matrix table computed during for averages, you will see the pattern for the cube response for Y.



Overview

The Analysis of Variance (ANOVA) table 1, page 12 did not indicate which specific mean effect or effects and interactions are statistically significant, etc. The ANOVA uses the ***F-distribution*** method. This method employs the regression approach with adjusted mean sum of squares (MS), sum of squares (SS), and means sum of squares due to error (MSE) for analysis. However, the ***t-distribution*** in table 2, page 13, shows which mean effects and interactions are statistically significant or statistically insignificant. The t-distribution will enable the experimenter to take a closer look at the mean effects and interactions in order to make decisions. The General Linear Model (GLM), which employs ANOVA was also used for the ***Reduced Model*** where important factors and interactions were entered into Minitab. This resulted in an ANOVA Table showing the significance of important factors, etc. See table 3, page 14.

(continued)

Table 1

Analysis of Variance for Response (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	1.19537	1.19537	0.39846	17.60	0.001
2-Way Interactions	3	0.35737	0.35737	0.11912	5.26	0.027
3-Way Interactions	1	0.01051	0.01051	0.01051	0.46	0.515
Residual Error	8	0.18115	0.18115	0.02264		
Pure Error	8	0.18115	0.18115	0.02264		
Total	15	1.74439				

(continued)

Table 2

Fractional Factorial Fit: Response-Y versus X1, X2 & X3

These values were hand computed by the team during training.

Estimated Effects and Coefficients for Response (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		1.7856	0.03762	47.47	0.000
X1-Car Type	-0.1562	-0.0781	0.03762	-2.08	0.071
X2-LH	-0.4337	-0.2169	0.03762	-5.76	0.000
X3-Track	0.2937	**0.1469**	**0.03762**	**3.90**	**0.005**
X1-Car T*X2-LH	**0.2688**	**0.1344**	**0.03762**	**3.57**	0.007
X1-Car T*X3-Track	0.0912	0.0456	0.03762	1.21	0.260
X2-LH*X3-Track	0.0938	0.0469	0.03762	1.25	0.248
X1-Car T*X2-LH*X3-Track	0.0513	0.0256	0.03762	0.68	0.515

Note: See main effects computation on exercise sheet. The effect column should be close to the values computed in the class exercise. The largest effect = - 0.4337. Wherever p-value is less than 0.10, the effect is statistically significant because testing is at the 10% level of significance. You may note that factor X1-Car Type indicates a p-value = 0.071 < 0.10, which is a rather weak effect. Additionally, the only interaction is between car type & launch height (X1*X2) factors as indicated on the interaction graph and in this table with p-value =0.007 < 0.10.

Table 3

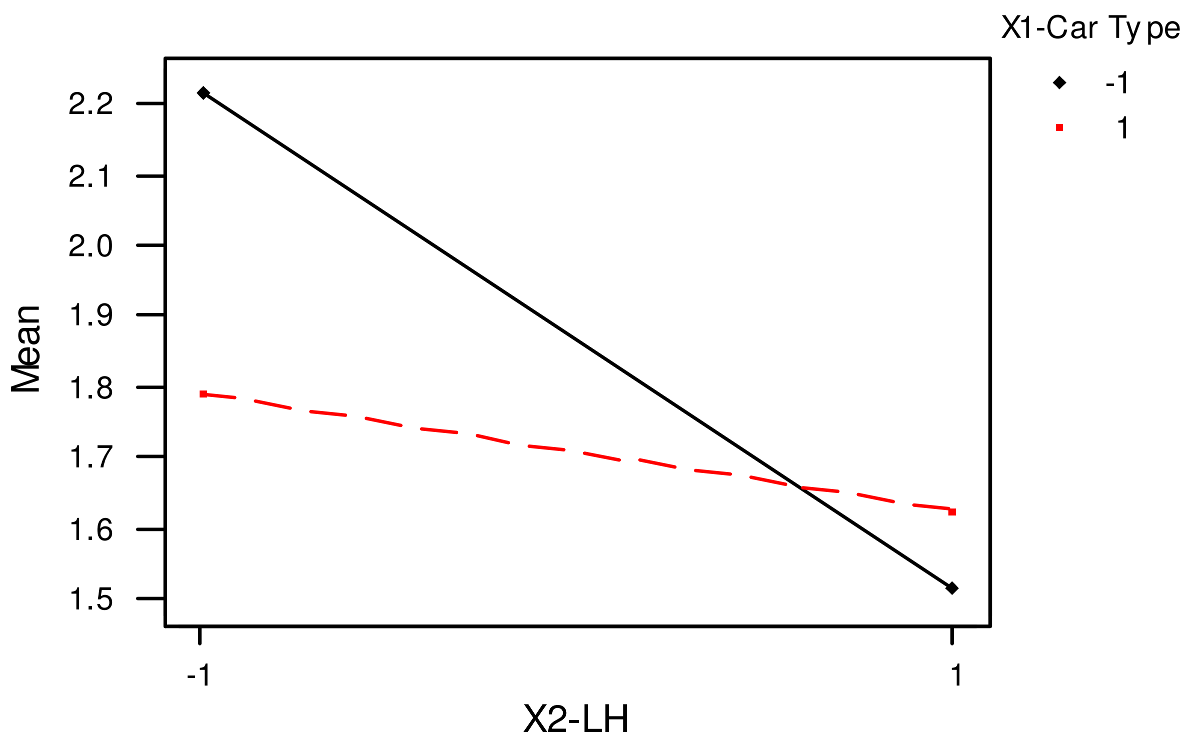
Analysis of Variance for Response, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
X1-Car Type	1	0.09766	0.09766	0.09766	4.13	0.067
X2-LH	1	0.75256	0.75256	0.75256	31.82	0.000
X3-Track Conf.	1	0.34516	0.34516	0.34516	14.60	0.003
X1-Car T*X2-LH	1	0.28891	0.28891	0.28891	12.22	0.005
Error	11	0.26012	0.26012	0.02365		
Total	15	1.74439				

Reduced Model Interaction

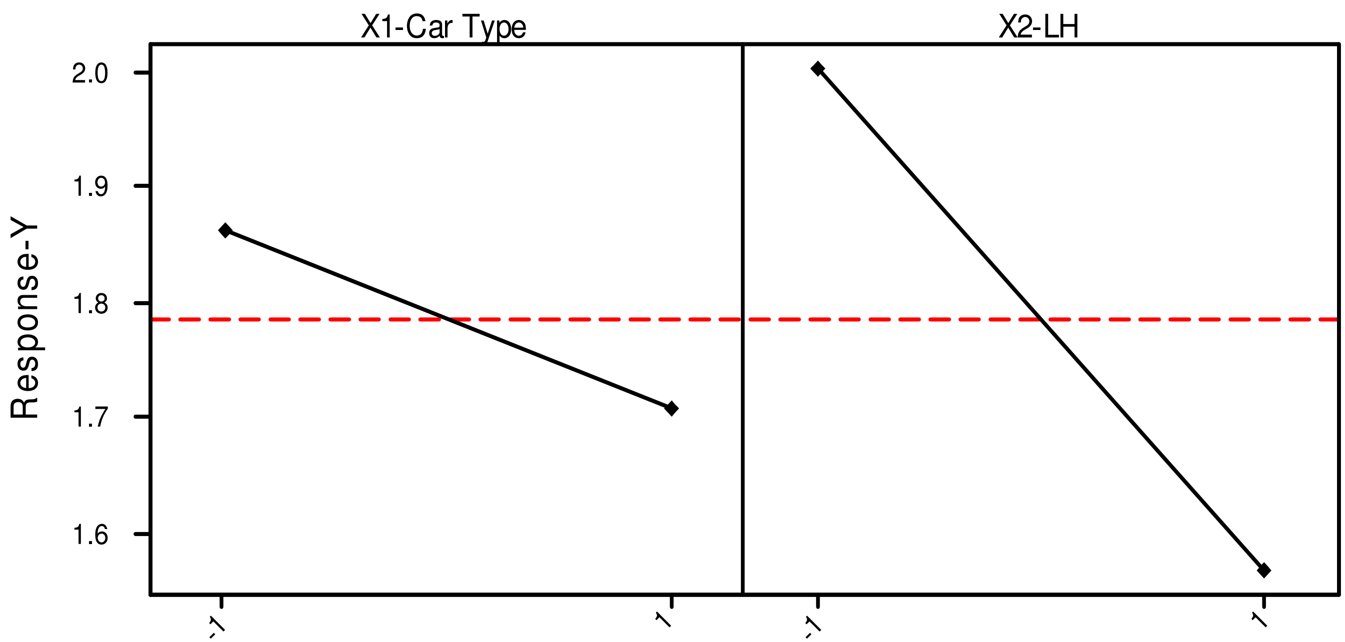
(continued)

Interaction Plot - LS Means for Response-Y



(continued)

Main Effects Plot - LS Means for Response-Y



Mathematical Model (continued)

$$Speeed = Avg. + \frac{Effect_{x_1}}{2} * x_1 + \frac{Effect_{x_2}}{2} * x_2 + \frac{Effect_{x_3}}{2} * x_3 + \frac{Effect_{x_1 x_2}}{2} * x_1 * x_2$$

$$Speed = 1.7856 - 0.0781\left(Car_{x_1}\right) - 0.2169\left(LH_{x_2}\right) + 0.1469\left(Track_{x_3}\right) + 0.1344\left(Car_{x_1} * LH_{x_2}\right)$$

Estimate of *y*

$$\hat{y} = \hat{\beta}_o + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_3 + \hat{\beta}_{1,2} x_1 x_2$$

Response surface "*y*"

Note that beta-hat is one-half corresponding factor coefficient effect estimates as shown in the top equation. The reason is that the regression coefficient is one-half the effect estimate. This is because the regression coefficient measures the effect of unit change in ***"x"*** on the mean of ***"y"***, and the effect estimate is based on a two-unit change from -1 to +1. Hence, this is a two level DOE.

An Academe Perspective:

The next two pages are tutorial in nature with the objective of adding some breadth and depth to test for significance and/or the term statistically significant or statistically insignificant.

Two examples are used, one for a ***t-distribution*** and one for a ***F-distribution***. Both models were used in analyzing the training results form the DOE exercise. The factor X_3- Track configuration was used for both models on the following pages.

What is a p-value?

The p-value represents the probability of making the mistake of getting a type one error or rejecting the null hypothesis when it is true. Basically, this means that the smallest value of alpha (α) for which the hypothesis would be rejected or deemed that the difference is statistically significant. The ***p-value*** derived from the test statistic which is computed from sample data.

For example, when the ***p-value*** **>** α**,** the conclusion is statistically insignificant and when the ***p-value*** **<** α, the conclusion is statistically significant. One can be a subset of the other and vice versa.

Design of Experiment — Test for Significance

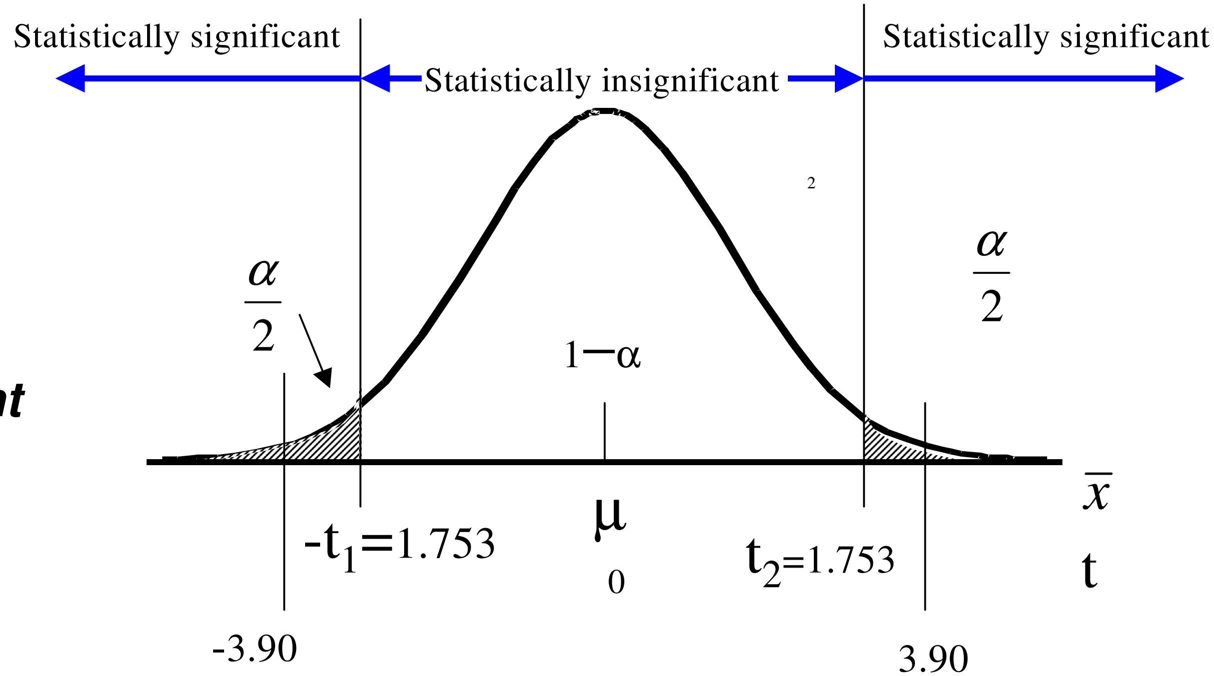
For example, see Table 2, Factor X_3 Track Configuration ***Using the t--distribution***

Test for significance

$$H_o: \beta_3 = 0$$
$$H_a: \beta_3 \neq 0$$
$$\alpha = 0.10$$

Let β = coefficient

$$E(\bar{x}) = \mu$$



Test statistic

$$t = \frac{b_3 - \beta_3}{s_{b3}} = \frac{Coef_{x_3}}{SE\ Coef} = \frac{0.1469}{0.03762} = 3.886 \approx 3.90$$

Let TS '*t*' be represented by the dummy variable *q*.

$$P - Value = \int_{q_2}^{\infty} f(q)\,dq + \int_{-q_1}^{-\infty} f(q)\,dq$$

Critical value

$$t_{\alpha/2, df} = t_{0.05, 15} = 1.753$$

$$\therefore\ P - Value = \int_{3.90}^{\infty} f(q)\,dq + \int_{-3.90}^{-\infty} f(q)\,dq = 0.005$$

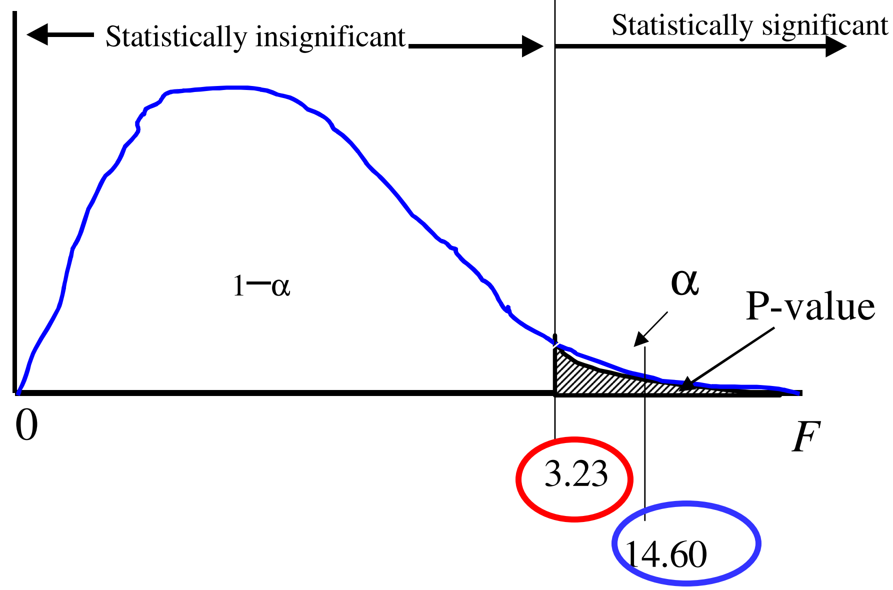
Note how nicely these computations agree with Minitab version in Table 2

For Example, see Table 3, Factor X_3, Track Configuration

Using the F--distribution

Test of significance

$$H_o: \beta_3 = 0$$
$$H_a: \beta_3 \neq 0$$
$$\alpha = 0.10$$



Critical value

$$F_{\alpha, df_1, df_2} = F_{0.10,1,11} = 3.23$$

$$P - Value = \int_0^\infty f(F)\, dF$$

Test statistic

$$F = \frac{MS_{x_3}}{MSE} = \frac{0.34516}{0.02365} = 14.5945 \approx 14.60$$

$$\therefore P - Value = \int_{14.6}^\infty f(F)\, dF = 0.003$$

MS= Mean Sum of Squares

MSE = Mean Sum of Squares due to Error

Note how nicely these computations agree with Minitab version in Table 3