WP-2

Statistical Sampling in a Production Process

"A Practical Guide to Statistical Sampling"

A White Paper

by

David C. Wilson, MSEE
Wilson Consulting Services, LLC
December 23, 2004

ABSTRACT

This paper provides practical exercises for determining statistical sampling. The intended audiences are manufacturing, quality, receiving, and test engineering interested in how to determine a reasonable statistical sampling plan. This information is being provided as a result of the author's analyzing production inspection data from a manufacturing facility.

The paper presents several scenarios and examples on determining a sampling plan using real production data from the Northford Plant Facility.

The lot size, N, the sample size, n, and the acceptance number, c, uniquely determine a sampling plan. When N is large, inferential statistics is most widely used as in this case, uniquely n and c. The simplest way is to get n and c for a given percent defectives and matching alpha and beta risks. There are many computer programs that will do the calculations. The introduction section in this paper defines the various mathematical methods used to determine sample size, defectives, probabilities, etc.

Table of Contents

1.0: Introduction

Objective:

To determine the outputs of various statistical methods in order to develop statistical inspection sampling plans for various manufacturing process steps.

Information Source

- March and April 2003 Monthly Production Quality Reports
- April 2002 Monthly Production Quality Report

Analysis Resources:

- Mintab
- Excel
- Distribution tables

Statistical Methodologies

- Binomial distribution
- Poisson Distribution
- Exact method to the binomial distribution

Note: *This paper was first drafted in May 5, 2003. Extensive revisions have been made since that time.*

Reference: *Applied Reliability by Paul A. Tobias & David C. Trindade,*
Publisher: Chapman & Hall/CRC, Second Edition, 1995.

continued

Cumulative Binomial Model

The binomial distribution is a special case of a discrete distribution.

1. It has only two outcomes are possible, e.g.. pass/fail go/no go, etc.
2. There are a fixed number of (n) trials.
3. There exist a fixed number of probability, p, of success from trial to trial.
4. The outcomes are independent from trial to trial.

$$P(X \le x) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x}$$

Cumulative Poisson Model

The Poisson distribution is a special case of a discrete distribution.

1. It has only two outcomes are possible, e.g.. failures in a given time or defects in a length of wire, wafers, etc.
2. The outcomes are independent from trial to trial.
3. Can be considered an extension of the binomial when n is large.
4. It provides a good approximation to the bimanual distribution, when p or q is small and n is large (Definition of small p (i.e., $p < 0.10$), where λ is the average failure rate *($\lambda = np$)*

$$P(X \le x) = \sum_{x=0}^{c} \frac{\lambda^x e^{-\lambda}}{x!}, x = 0,1,2,...,$$

2.0 Summary

I. Production Data

Binomial distribution

In this paper binomial probabilities were developed for several process steps in response to an inquiry To the author on how to audit and/or sampling various process steps rather do a 100% inspection at each step in the process. A 95% confidence bound was used, which means that a sample of products that Indicates a probability that exceeds 0.95 would be rejected and require a 100% inspection. These tables are not a final version, rather they have been generated to provoke a thoughtful discussion on how to come-up with a sampling plan that will satisfy the process steps. It is important to minimize Types I (α) and Type II (β) errors.

For example: Table 1, sample size n = 1000 and p = 0.0006, which has a 0.06% defective rate. The 0.06% is a historic monthly failure rate. If the assumption that an acceptance "c" such that at least 95% of the time a lot is accepted if the true percent defective level is 0.06% or less. Therefore, the risk of Type I error (α) is 5%. The cumulative binomial probabilities for 0, 1, 2, 3, etc., up to "c" failures, for $n = 1000$ and $p = 0.0006$ *are* associated with the probability of getting "c" or less exceeds 95% or 0.95. This means that 95% of the time, the lot with the historic 0.06% defectives or less will be accepted because"c" failures or less are expected 95% for a sample size n drawn from a large lot with this historic failure rate. For n = 1000, the the number of acceptable defects is 1 or less as shown in table 1.

continued

II. Operating Characteristic (OC) Curves

The OC curves provide a given sample size and acceptance number. The percent defective proportion value are read from the inside of the table at the intersection of the probability of acceptance and sample size. For example, c = 3 failures has probability of acceptance of 0.80 or 80% and a defective rate of 4.63% for n = 50. See Tables 14 & figure 1. Please note that the higher the probability of acceptance the smaller the defect rate.

3.0 QA Data

Example from April Monthly Quality Report

See Sampling Tables for acceptance/rejection of a lot.

Process Steps:

Automatic SMT (1st Piece / Audits)

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	9881	15,245	11678
Total Pieces Rejected	7	10	5

Pre-Wave Inspection (1st Piece)

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	334	578	447
Total Pieces Rejected	5	9	4

Post-Wave Inspection

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	316	564	585
Total Pieces Rejected	6	9	13

Example from April 2003 Monthly Quality Report – cont'd

Process Steps (continued)

Functional

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	5951	9289	11,686
Total Pieces Rejected	109	186	217

Pre-Post Burn-in

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	1,100	1309	5258
Total Pieces Rejected	54	31	54

Final Visual Inspection

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	8261	10,801	8,898
Total Pieces Rejected	70	124	130

Example from April 2003 Monthly Quality Report – cont'd

Process Steps (continued)

Finished Goods Audit (Post -Pack)

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	44	154	373
Total Pieces Rejected	6	1	8

Metal Fabrication

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	621	735	605
Total Pieces Rejected	5	11	2

Shipping Audits

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	366	559	605
Total Pieces Rejected	1	3	10

Example from April 2003 Monthly Quality Report – cont'd

Process Steps (continued)

Stockroom Audits

Description	April 2003	March 2003	April 2002
Total Pieces Inspected	677	762	163
Total Pieces Rejected	17	17	10

4.0 Lot Size vs. Acceptance

For a lot to be acceptable, the probability of ‘c’ occurring must be equal or less than 0.95. The historic proportion was determined by the average of the three months of defective data.

Sampling Tables

Process Step Auto SMT

n = 1000 and p = 0.0006

c	$P(X \leq c)$
5	0.99996
4	0.99961
3	0.99666
2	0.97693
1	0.87814
0	0.54871

Process Step Auto SMT

n = 3000 and p = 0.0006

c	$P(X \leq c)$
5	0.98965
4	0.96364
3	0.89135
2	0.73064
1	0.46277
0	0.16521

Table 1

Process Step Auto SMT

n = 2000 and p = 0.0006

c	$P(X \leq c)$
5	0.99851
4	0.99228
3	0.96628
2	0.87954
1	0.66261
0	0.30109

Process Step Auto SMT

n = 5000 and p = 0.0006

c	$P(X \leq c)$
5	0.91614
4	0.81531
3	0.64723
2	0.42312
1	0.19906
0	0.04974

Table 2

Sampling Tables – cont'd

Derived from the Poisson Model

This is the only page, which used the Poisson distribution in its Probabilistic Tables, all others estimates used binomial distributions.

Process Step Auto SMT $\lambda = 0.6$

c	$P(X \le c)$
5	0.99996
4	0.99961
3	0.99664
2	0.97689
1	0.87810
0	0.54881

Process Step Auto SMT $\lambda = 1.8$

c	$P(X \le c)$
5	0.98962
4	0.96359
3	0.89129
2	0.73062
1	0.46284
0	0.16530

Table 3

Process Step Auto SMT $\lambda = 1.2$

c	$P(X \le c)$
5	0.99850
4	0.99225
3	0.96623
2	0.87949
1	0.66263
0	0.30119

Process Step Auto SMT $\lambda = 3.0$

c	$P(X \le c)$
5	0.91608
4	0.81526
3	0.64723
2	0.42319
1	0.19915
0	0.04979

Table 4

Pre-wave inspection – sampling tables

Pre-Wave Inspection (1st Piece)
n = 100 and p = 0.013

c	$P(X \le c)$
9	1.00000
8	0.99999
7	0.99995
6	0.99965
5	0.99797
4	0.98990
3	0.9580
2	0.8582
1	0.6261
0	0.2702

Pre-Wave Inspection (1st Piece)
n = 300 and p = 0.013

c	$P(X \le c)$
9	0.99349
8	0.98218
7	0.95571
6	0.90085
5	0.80166
4	0.64850
3	0.452068
2	0.251213
1	0.097695
0	0.019731

Table 5

Pre-Wave Inspection (1st Piece)
n = 200 and p = 0.013

c	$P(X \le c)$
9	0.99967
8	0.99865
7	0.99500
6	0.98353
5	0.95212
4	0.87872
3	0.7366
2	0.5174
1	0.265366
0	0.073018

Pre-Wave Inspection (1st Piece)
n = 400 and p = 0.013

c	$P(X \le c)$
9	0.96137
8	0.91941
7	0.84625
6	0.73319
5	0.58069
4	0.40481
3	0.236206
2	0.107232
1	0.033421
0	0.005332

Table 6

Post-wave inspection

Post-Wave Inspection

n = 100 and p = 0.019

c	$P(X \le c)$
9	0.99998
8	0.99987
7	0.99933
6	0.99693
5	0.98769
4	0.95755
3	0.8765
2	0.7040
1	0.4313
0	0.1469

Post-Wave Inspection

n = 300 and p = 0.019

c	$P(X \le c)$
9	0.93703
8	0.87866
7	0.78578
6	0.65483
5	0.49386
4	0.32481
3	0.177381
2	0.07486
1	0.021571
0	0.003167

Table 7

Post-Wave Inspection

n = 200 and p = 0.013

c	$P(X \le c)$
9	0.99470
8	0.98497
7	0.96144
6	0.91109
5	0.81727
4	0.66823
3	0.4719
2	0.2661
1	0.105112
0	0.021568

Post-Wave Inspection

n = 400 and p = 0.019

c	$P(X \le c)$
9	0.76643
8	0.64873
7	0.50921
6	0.36256
5	0.22805
4	0.12255
3	0.053772
2	0.017993
1	0.004069
0	0.000465

Table 8

Functional Test

Functional

n = 3000 and p = 0.019

c	$P(X \le c)$
200	1.00000
175	1.00000
150	1.00000
125	1.00000
100	1.00000
90	0.99998
60	0.6861
50	0.1937
40	0.0106
0	0.0000

Functional

n = 7000 and p = 0.019

c	$P(X \le c)$
200	1.00000
175	0.99982
150	0.93506
125	0.25830
100	0.00155
90	0.00004
60	0.00000
50	0.00000
40	0.00000
0	0.00000

Table 9

Functional

n = 5000 and p = 0.019

c	$P(X \le c)$
200	1.00000
175	1.00000
150	1.00000
125	0.99877
100	0.71936
90	0.32526
60	0.0001
50	0.0000
40	0.0000
0	0.0000

Functional

n = 9000 and p = 0.019

c	$P(X \le c)$
200	0.98712
175	0.63994
150	0.05449
125	0.00012
100	0.00000
90	0.00000
60	0.00000
50	0.00000
40	0.00000
0	0.00000

Table 10

Pre- / Post Burn-in

Pre / Post Burn-in

n = 500 and p = 0.028

c	$P(X \le c)$
50	1.00000
45	1.00000
40	1.00000
35	1.00000
30	0.99996
25	0.99775
20	0.9545
15	0.6708
10	0.1719
0	0.0000

Pre / Post Burn-in

n =1000 and p = 0.028

c	$P(X \le c)$
50	0.99995
45	0.99906
40	0.98862
35	0.92079
30	0.69229
25	0.32434
20	0.06994
15	0.00495
10	0.00007
0	0.00000

Table 11

Pre / Post Burn-in

n = 800 and p = 0.028

c	$P(X \le c)$
50	1.00000
45	0.99999
40	0.99979
35	0.99563
30	0.95344
25	0.75282
20	0.3525
15	0.0633
10	0.00254
0	0.00000

Pre / Post Burn-in

n = 1200 and p = 0.028

c	$P(X \le c)$
50	0.99730
45	0.97746
40	0.88436
35	0.63940
30	0.30057
25	0.07351
20	0.00740
15	0.00023
10	0.00000
0	0.00000

Table 12

Final Visual Inspection

Final Visual Inspection

n = 5000 and p = 0.011

c	$P(X \leq c)$
120	1.00000
100	1.00000
80	0.99943
70	0.97908
65	0.91978
60	0.77509
50	0.2755
40	0.0207
20	0.0000
10	0.0000

Final Visual Inspection

n =9000 and p = 0.011

c	$P(X \leq c)$
120	0.98288
100	0.56658
80	0.02774
70	0.00127
65	0.00017
60	0.00002
50	0.00000
40	0.00000
20	0.00000
10	0.00000

Table 13

Final Visual Inspection

n = 8000 and p = 0.011

c	$P(X \leq c)$
120	0.99954
100	0.90780
80	0.21247
70	0.02712
65	0.00608
60	0.00095
50	0.00001
40	0.00000
20	0.00000
10	0.00000

Final Visual Inspection

n = 10000 and p = 0.011

c	$P(X \leq c)$
120	0.84300
100	0.18184
80	0.00157
70	0.00003
65	0.00000
60	0.00000
50	0.00000
40	0.00000
20	0.00000
10	0.00000

Table 14

Finish Goods Audit & Final Inspection (post pack)

Finished Goods Audit (post pack)

n = 50 and p = 0.055

c	$P(X \le c)$
10	0.99993
9	0.99966
8	0.99852
7	0.99435
6	0.98099
5	0.94449
4	0.86087
3	0.70469
1	0.23109
0	0.05910

Finished Goods Audit (post pack)

n =100 and p = 0.055

c	$P(X \le c)$
10	0.97827
9	0.95130
8	0.90039
7	0.81482
6	0.68834
5	0.52650
4	0.35089
3	0.19374
1	0.02382
0	0.00349

Table 15

Final Visual Inspection (post pack)

n = 75 and p = 0.055

c	$P(X \le c)$
10	0.99739
9	0.99208
8	0.97825
7	0.94634
6	0.88183
5	0.76939
4	0.60378
3	0.40341
1	0.07708
0	0.01437

Final Visual Inspection (post pack)

n = 110 and p = 0.055

c	$P(X \le c)$
10	0.95997
9	0.91848
8	0.84790
7	0.74089
6	0.59808
5	0.43294
4	0.27079
3	0.13938
1	0.01468
0	0.00198

Table 16

Metal Fabrication

Metal Fabrication

n = 100 and p = 0.0088

c	$P(X \leq c)$
10	1.00000
9	1.00000
8	1.00000
7	1.00000
6	0.99997
5	0.99973
4	0.9980
3	0.9880
1	0.7800
0	0.4132

Metal Fabrication

n =400 and p = 0.0088

c	$P(X \leq c)$
10	0.99900
9	0.99671
8	0.99012
7	0.97309
6	0.93402
5	0.85583
4	0.722067
3	0.531829
1	0.132632
0	0.029142

Table 17

Metal Fabrication

n = 300 and p = 0.0088

c	$P(X \leq c)$
10	1.00000
9	0.99999
8	0.99992
7	0.99955
6	0.99786
5	0.99097
4	0.9671
3	0.8985
1	0.47383
0	0.17071

Metal Fabrication

n = 500 and p = 0.0088

c	$P(X \leq c)$
10	0.99454
9	0.98553
8	0.96488
7	0.92231
6	0.84452
5	0.72036
4	0.550845
3	0.358369
1	0.065489
0	0.012041

Table 18

Shipping Audits

Shipping Audits

n = 100 and p = 0.0082

c	$P(X \leq c)$
10	1.00000
9	1.00000
8	1.00000
7	1.00000
6	0.99998
5	0.99981
4	0.99854
3	0.99049
2	0.95038
1	0.80186

Shipping Audits

n =400 and p = 0.0082

c	$P(X \leq c)$
10	0.99943
9	0.99800
8	0.99358
7	0.98130
6	0.95108
5	0.88612
4	0.76677
3	0.58451
2	0.36239
1	0.15989

Table 19

Shipping Audits

n = 300 and p = 0.0082

c	$P(X \leq c)$
10	0.99995
9	0.99978
8	0.99904
7	0.99631
6	0.98728
5	0.96127
4	0.89729
3	0.76657
2	0.55363
1	0.29434

Shipping Audits

n = 500 and p = 0.0082

c	$P(X \leq c)$
10	0.99674
9	0.99076
8	0.97603
7	0.94345
6	0.87951
5	0.76991
4	0.60923
3	0.41332
2	0.22261
1	0.08366

Table 20

Stockroom Audits

Stockroom Audits

n = 100 and p = 0.036

c	$P(X \le c)$
15	1.00000
12	0.99994
10	0.99902
9	0.99669
8	0.98982
7	0.97182
6	0.93036
5	0.84769
3	0.51288
1	0.12105

Stock Room Audits

n =175 and p = 0.036

c	$P(X \le c)$
15	0.99935
12	0.98877
10	0.94708
9	0.89781
8	0.81833
7	0.70362
6	0.55737
5	0.39514
3	0.12179
1	0.01232

Table 21

Stockroom Audits

n = 150 and p = 0.036

c	$P(X \le c)$
15	0.99989
12	0.99683
10	0.97965
9	0.95443
8	0.90652
7	0.82522
6	0.70342
5	0.54487
3	0.20809
1	0.02699

Stockroom Audits

n = 200 and p = 0.036

c	$P(X \le c)$
15	0.99741
12	0.96996
10	0.89064
9	0.81322
8	0.70466
7	0.56841
6	0.41717
5	0.27104
3	0.06840
1	0.00554

Table 22

5.0 Operating Characteristic (OC) Curves

As the acceptance number c is reduced from 5, 4, 3, 2, 1, etc., the consumer's risk (Type II error) is increased and a Type 1 error (producer's risk) is reduced. The converse is true, i.e., if Type II risk is reduced then the Type I increases. The objective should be to reach a balance on the acceptable number as to minimize both risks.

	Various Probabilities of c for n = 50		
Lot percent lot defective *(p)*	$P(X \leq 1)$	$P(X \leq 3)$	$P(X \leq 5)$
0.00	1.0000	1.0000	1.0000
0.01	0.9105	0.9984	0.9999
0.02	0.7358	0.9822	0.9995
0.03	0.5530	0.9372	0.9963
0.04	0.4004	0.8609	0.9856
0.05	0.2794	0.7604	0.9622
0.06	0.1900	0.6473	0.9223
0.07	0.1265	0.5327	0.8650
0.08	0.0827	0.4253	0.7919
0.09	0.0532	0.3303	0.7072
0.10	0.0339	0.2503	0.6162
0.12	0.0131	0.1345	0.4353
0.15	0.0029	0.0460	0.2193
0.18	0.0006	0.0137	0.0928
0.20	0.0002	0.0056	0.0408
0.25	0.0000	0.0005	0.0070

Table 23

Tables of various sample sizes of OC curves – cont'd

The information in these abbreviated tables were derived from Minitab and represents the OC curves concept. Hence: defect rate decreases as the probability of acceptance increases.

Defect: c = 0 — **Probability of Acceptance**

Sample Size	0.10	0.80	0.90	0.95
10	20.5680	2.2068	1.0490	0.5117
25	8.7990	0.8870	0.4206	0.2050
50	4.5008	0.4453	0.2105	0.1025
100	2.2763	0.2229	0.1054	0.0513
500	0.4595	0.0446	0.0211	0.0103
1000	0.2300	0.0223	0.0105	0.0051
5000	0.0461	0.0045	0.0021	0.0010
10000	0.0230	0.0022	0.0011	0.0005

Table 24

Defect: c = 3 — **Probability of Acceptance**

Sample Size	0.10	0.80	0.90	0.95
10	33.6850	23.9450	18.7570	15.0030
25	14.6868	9.3259	7.1670	5.6570
50	7.5590	4.6280	3.5348	2.7788
100	3.8340	2.3060	1.7559	1.3777
500	0.7757	0.4597	0.3494	0.2738
1000	0.3885	0.2298	0.1746	0.1368
5000	0.0778	0.0459	0.0349	0.0273
10000	0.0389	0.0230	0.0175	0.0137

Table 25

Graph of OC Curves from table 23

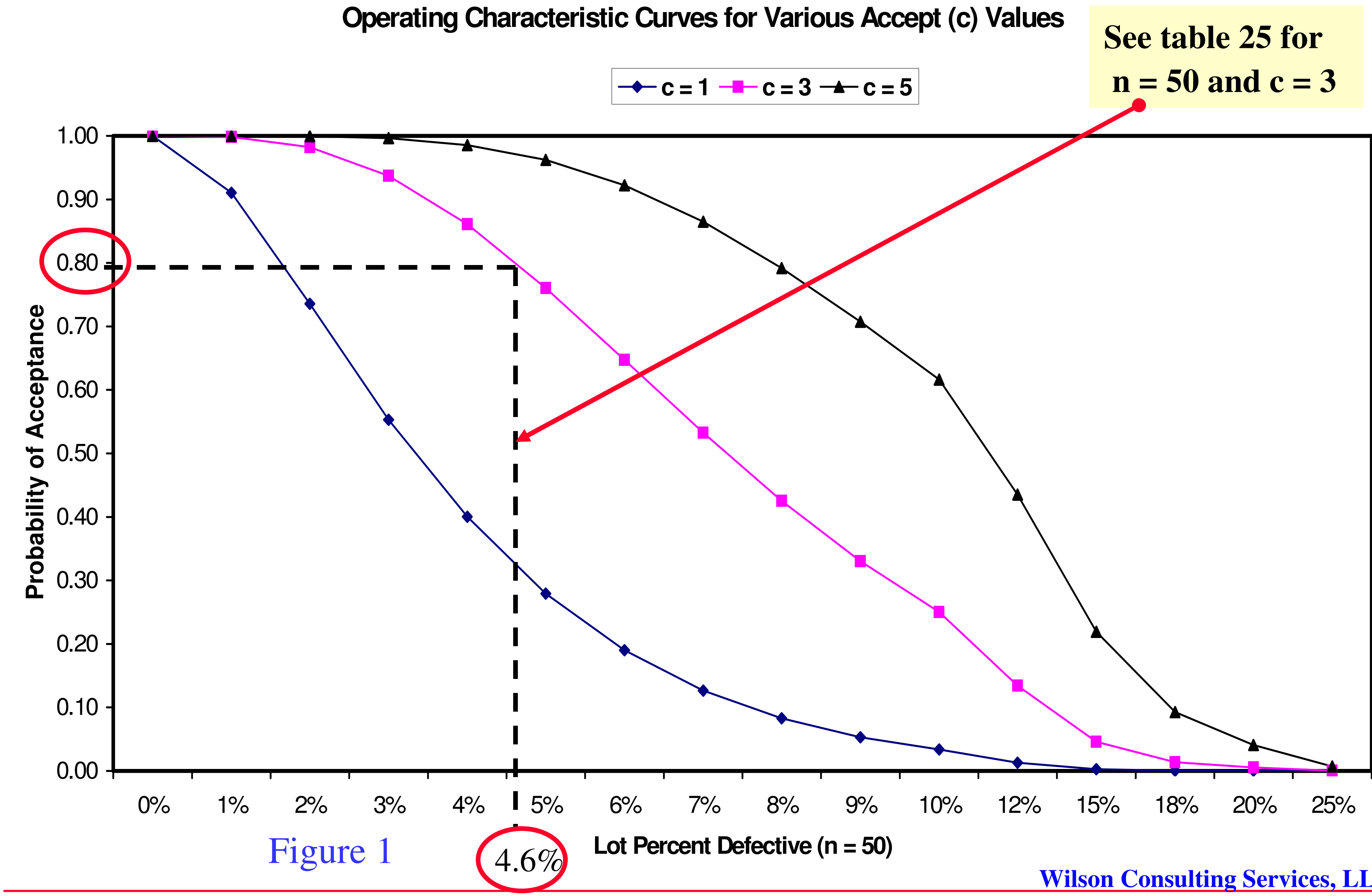

Figure 1